Example DPE:

Cylindrical vessel loaded with external pressure

Given:

Cylindrical vessel loaded with external pressure p_e= 0.1 MPa, with outside diameter D_e = 1500 mm and length L = 1600 mm, steel with yield point σ_K = 210 MPa and modulus of elasticity E = 210 * GPa. Corrosion allowance is c = 1 mm.

Task:

Specify needed wall thickness s = ?.

By reason of simplification we will study a cylinder without effects of transition stress near bottoms and footings.

Specification of wall thickness according Czech standard ČSN 690010, part 4.5

a) **Calculating wall thickess is approximately specified from following equation**
(part 3.2.1)

$$s_R = Max\left\{K_2 * D * 10^{-2} ; \frac{1,1 * p * D}{2 * [\sigma_D]}\right\}$$

Realized wall thickness is

$$s \geq s_R + c$$

For $K_3 = L / D = 1600/1500 = 1.07$

and $K_1 = (n_U*p) / (2.4*10^{-6}*E) = (2.4*0.1) / (2.4*10^{-6}*2.1*10^5) = 0.48$

Where $n_U = 2.4$ ..... safety coefficient against a stability loss in an elastic region (according part 4.2)

we specify from diagram fig.3: $K_2 = 0.32$

Then it is

$s'_R = K_2 * D * 10^{-2} = 0.32*1500*10^{-2} = 4.80$ mm

$s''_R = 1.1*p*D / 2*[\sigma_D] = 1.1*0.1*1500 / 2*140 = 0.59$ mm

$s_R = \text{Max}\ \{4.80; 0.59\} = 4.8$ mm

$s \geq 4.8 + 1.0 = 5.8$ mm

Realized wall thickness s = 7 mm is suitable.

b) <u>Maximal allowed external pressure</u> is specified according čl. 3.2.2.

$$[p] = \frac{[p_P]}{\sqrt{1 + \left(\frac{[p_P]}{[p_E]}\right)^2}}$$

Where an allowed external overpressure in a plastic state is

$$[p_P] = \frac{2*[\sigma_D]*(s-c)}{D+(s-c)} = \frac{2*140*(7,0-1,0)}{1500+(7,0-1,0)} = 1,12MPa$$

and allowed external overpressure in elastic state is

$$[p_E] = \frac{20{,}8 * 10^{-6} * E}{n_U * B_1} * \frac{D}{L} * \left[\frac{100 * (s-c)}{D}\right]^2 * \sqrt{\frac{100 * (s-c)}{D}}$$

Where is

n_U = 2.4 .... safety coefficient against a stability loss in an elastic region for working conditions (according part 4.2)

$$B_1 = Min\left\{1{,}0; 9{,}45 * \frac{D}{L} * \sqrt{\frac{D}{100 * (s-c)}}\right\} =$$

$$B_1 = Min\left\{1{,}0; 9{,}45 * \frac{1500}{1600} * \sqrt{\frac{1500}{100 * (7-1)}}\right\}$$

$$B_1 = \{1{,}0; 14{,}0\} = 1{,}0$$

Then it will

$$[p_E] = \frac{20{,}8 * 10^{-6} * 2{,}1 * 10^5}{2{,}4 * 1{,}0} * \frac{1500}{1600} * \left[\frac{100 * (7-1)}{1500}\right]^2 * \sqrt{\frac{100 * (7-1)}{1500}} = 0{,}173 MPa$$

Maximal allowed external overpressure is

$$[p] = \frac{[1{,}12]}{\sqrt{1+\left(\frac{[1{,}12]}{[0{,}173]}\right)^2}} = 0{,}171\,MPa > p_{working} = 0{,}1\,MPa$$

From the point of view the external overpressure the cylinder shell is O.K.

c) Check of the allowed external overpressure according simplified equation with using of diagram fig.3

$$[p] = 2{,}4 * \frac{K_1 * 10^{-6} * E}{n_U}$$

where for parameters

$$K_2 = \frac{100*(s-c)}{D} = \frac{100*(7-1)}{1500} = 0{,}40$$

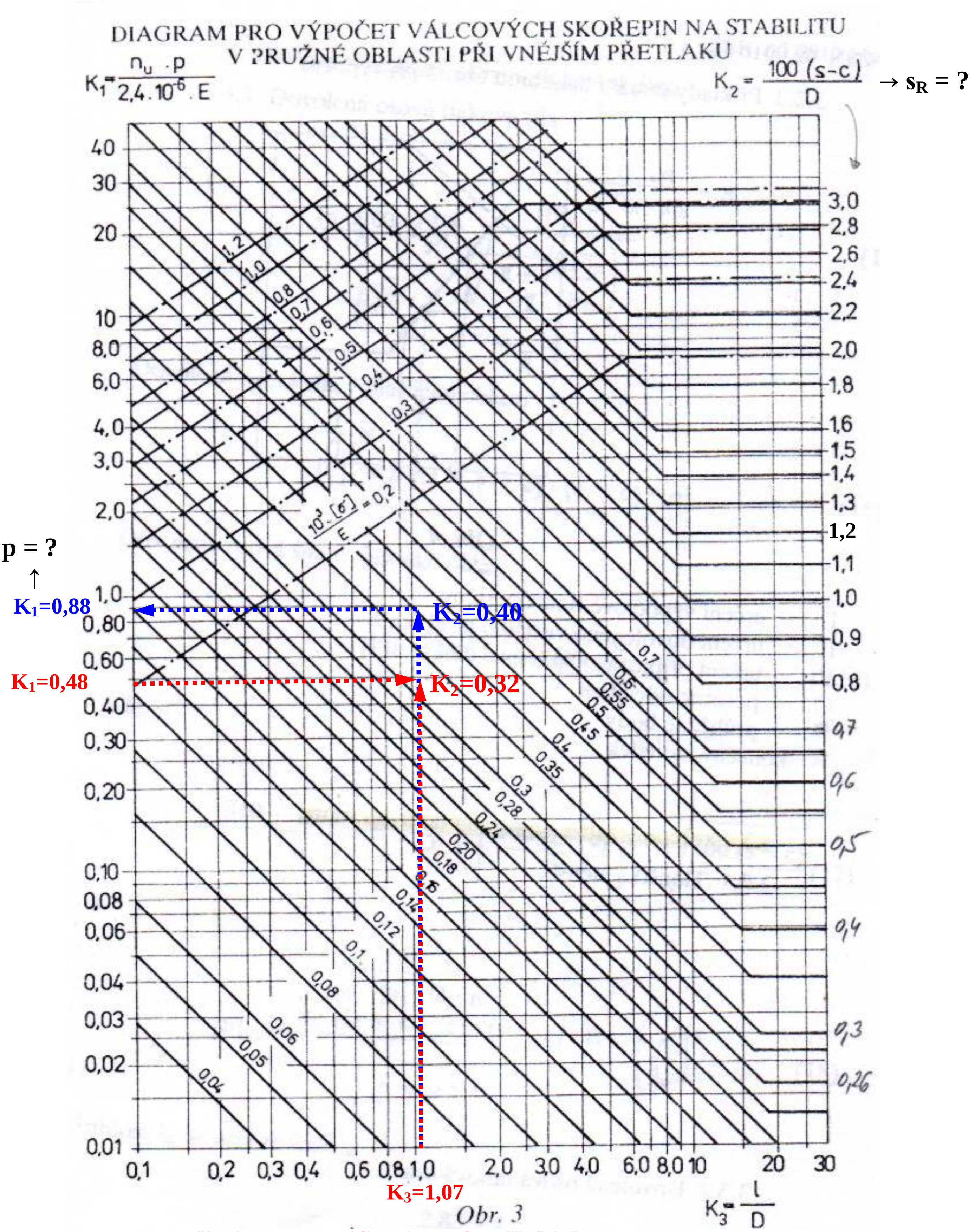
$$K_3 = \frac{L}{D} = \frac{1600}{1500} = 1{,}07$$

From diagram on fig.3 we can specify $K_1 = 0.88$. The point lies below the the dot-dash line (see the next page)

$$\frac{10^3 * [\sigma_D]}{E} = \frac{10^3 * 140}{2{,}1 * 10^5} = 0{,}67$$

and therefore we can use the above mentioned simplified equation.

Diagram for calculation of cylindrical shells for stability in elastic state for external overpressure



a) preliminary specification of wall thickness s

b) check of maximal alloweed external overpressure p_e

Then is the maximal allowed external overpressure

$$[p] = 2{,}4 * \frac{0{,}88 * 10^{-6} * 2{,}1 * 10^{5}}{2{,}4} = 0{,}185\ MPa$$

$$\mathbf{[p] > p_{working} = 0{,}1\ MPa}$$

Realized wall thickness s = 7 mm even from the viewpoint is O.K..

Other check calculation for the calculated wall thickness

In chap. a) the minimal calculating wall thickness s_R = 4,8 mm was specified. The wall thickness 7 – 1 = 6 mm allows too high pressure (overdesigned cylinder). Therefore we will do other check calculation for the minimal calculating thickness. The procedure is similar as it was in the part ad b) but instead value (s – c) the value s_R is used.

Allowed overpressure in the plastic state is

$$[p_P] = \frac{2 * [\sigma_D] * (s - c)}{D + (s - c)} = \frac{2 * 140 * 4{,}8}{1500 + 4{,}8} = 0{,}893\ MPa$$

Parameter B_1 = 1

Allowed overpressure in the elastic state is

$$[p_E] = \frac{20{,}8 * 10^{-6} * E}{n_U * B_1} * \frac{D}{L} * \left[\frac{100 * (s - c)}{D}\right]^2 * \sqrt{\frac{100 * (s - c)}{D}}$$

$$[p_E] = \frac{20{,}8 * 10^{-6} * 2{,}1 * 10^{5}}{2{,}4 * 1{,}0} * \frac{1500}{1600} * \left[\frac{100 * 4{,}8}{1500}\right]^2 * \sqrt{\frac{100 * 4{,}8}{1500}} = 0{,}099\ MPa$$

Maximal allowed external overpressure is then

$$[p]=\frac{[0,893]}{\sqrt{1+\left(\frac{[0,893]}{[0,099]}\right)^2}}=0,098\ MPa < p_{working}\ =0,100\ MPa$$

For the case is the wall thickness insufficient!

Calculation according an old Czech standard ČSN 690010

Basic wall thickness specified from strength point of view is calculated like for an internal overpressure but with a modified (higher) pressure.

$$s_R=\frac{p*D}{2*\sigma_D+p}*\beta$$

Where for L/D = 1600 / 1500 = 1.07 and σ_D / p = 140 / 0.1 = 1400 in a diagram 1 we can find a parameter β = 4.7. Then it is

$$s_R=\frac{0,1*1500}{2*140+0,1}*4,7=2,52\ mm$$

Then we specify a basic calculating wall thickness from the point of view of cylinder stability. We evaluate A = p / E * 10^6 = 0.1 * 10^6 / 2.1 *10^5 = 0.476

For A = 0.476 and L / D = 1.07 from diagram 2 we find a ratio

s_R / D ≈ 0.0033

Then is the basic calculated wall thickness specified from the stability point of view

s_R = 0.0033 * D = 0.0033 * 1500 = 4.95 mm

The value is practically the same like was according the new Czech standard ČSN 690010 when was the basic calculated wall thickness s_R = 4,8 mm, but after verification was slightly smaller (after recalculation was s_R = c. 4.9 mm - for maximal allowed external pressure 0.10 MPa).

Calculation according European standard EN 13445-3

Calculations are performed according chapter 8. Remember that the standard uses quite different symbols!

Firstly we must estimate an analyzed wall thickness e_a and for it a pressure P_y, is specified. At the pressure reaches a mean tangential stress in the shell the yield point (but in the standard is not specified how to estimate the value of e_a – it is a disadvantage of the standard EN!!).

$$P_y = \frac{\sigma_e * e_a}{R}$$

Where $\sigma_e = \sigma_D$ = 140 MPa; $e_a = s_R$ = s – c = 7 – 1 = 6 mm

R = (D – s) / 2 = (1500 – 7) / 2 = 746.5 mm (mean cylinder diameter)

Then is

$$P_y = \frac{140*6}{746,5} = 1,125\ MPa$$

Now we specify <u>theoretical pressure for the elastic loss of stability of the shell</u>

$$P_m = \frac{E*e_a*\varepsilon}{R}$$

Where for L / 2R = 1600 / 2*746.5 = 1.07 and

2R / e_a = 2*746.5 / 6 = 249

We can find in the diagram 8.5-3 parameter ε = 0.00035. Then is

$$P_m = \frac{2,1*10^5*6*0,00035}{746,5} = 0,59\ MPa$$

For ratio P_m / P_y = 0.59 / 1.125 = 0.524 we can specify from diagram 8.5-5 value of ratio P_r / P_y = 0.26.

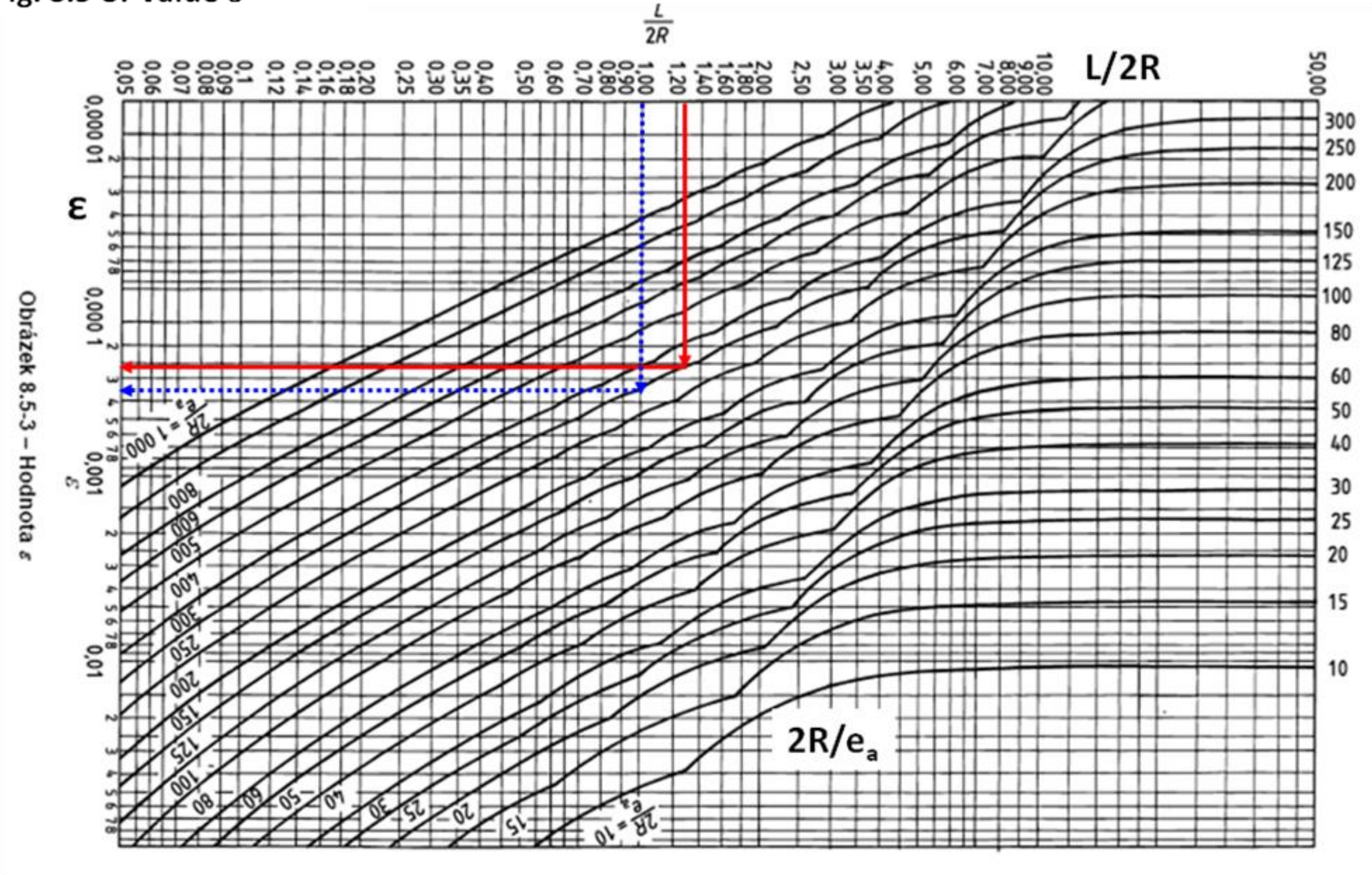
External overpressure must be lower then the ratio P_r / S, where S = 1.5 (safety coefficient for working conditions).

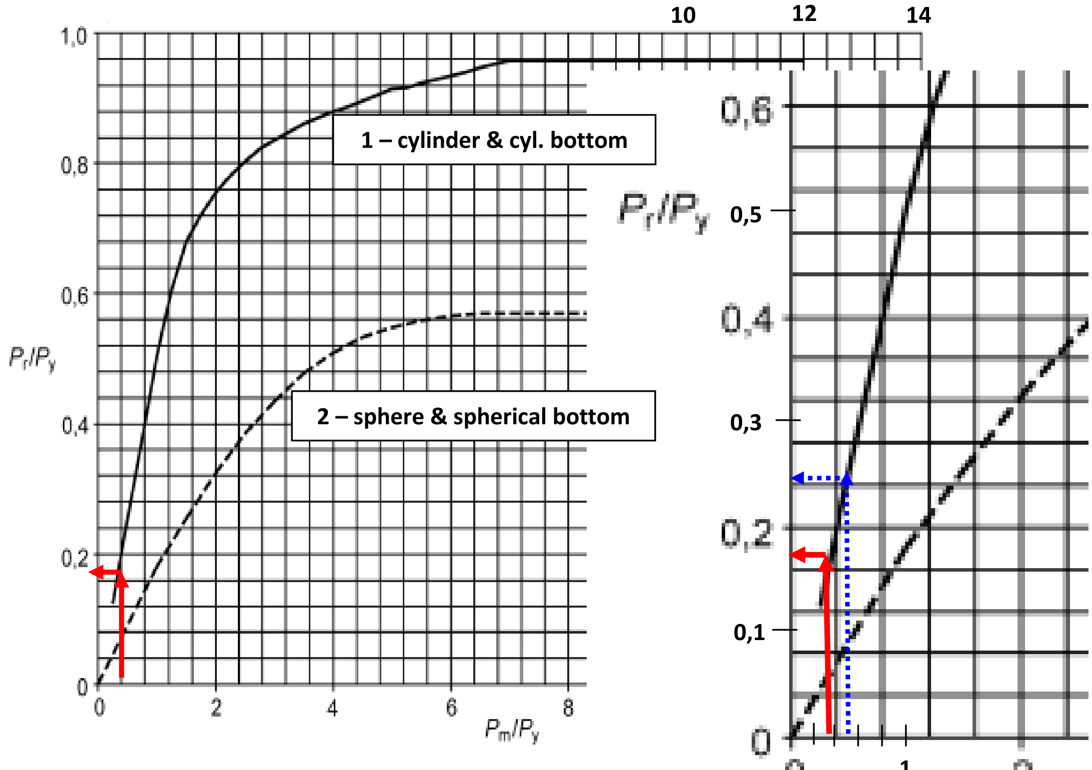
P_r / S = 0.26 / 1.5 = 0.170 MPa > P = 0.10 MPa

<u>Realized wall thickness is O.K.</u>

Note: According the new ČSN 690010 was allowed overpressure 0.171 MPa. From it follows that the EN is very slightly on the safety side.

Fig. 8.5-3: Value ε





1 - Válcové a kuželové pláště

P_m/P_y	0	0,25	0,5	0,75	1,0	1,25	1,5	1,75	2,0	2,25	2,5	2,75	3	3,25	3,5
P_r/P_y	0	0,125	0,251	0,375	0,5	0,605	0,68	0,72	0,755	0,78	0,803	0,822	0,836	0,849	0,861

P_m/P_y	3,75	4,0	4,25	4,5	4,75	5,0	5,25	5,5	5,75	6,0	6,25	6,5	6,75	≥ 7,0
P_r/P_y	0,87	0,879	0,887	0,896	0,905	0,914	0,917	0,923	0,929	0,935	0,941	0,947	0,953	0,959

2 - Kulové pláště a klenutá dna

P_m/P_y	0	0,5	1	1,5	2	2,5	3,0	3,5	4	4,5	5,0	5,5	6	≥ 6,5
P_r/P_y	0	0,09	0,18	0,255	0,324	0,386	0,435	0,479	0,51	0,533	0,548	0,565	0,567	0,57

Obrázek 8.5-5 – Hodnoty P_r/P_y jako funkce P_m/P_y

Fig. 8.5-5: Values Pr/Py as function of Pm/Py